Correlation stress testing of stock and credit portfolios

Natalie Packham

joint work with Fabian Woebbeking

CQF Institute

3 March 2021

Hochschule für Wirtschaft und Recht Berlin
Berlin School of Economics and Law

How do you obtain plausible correlation stress scenarios?

- Please type your answer / idea into the chat.
- **Wait** before you press 'Return'.

Overview

Motivation for correlation stress-testing

London Whale

- Background
- Correlation parameterisation
- Stress testing correlations

Generalised approach

- Principal ideas
- Bayesian factor selection
- Results

Conclusion

Overview

- **Correlation** lies at the heart of many financial applications: portfolio risk-management, diversification, hedging.
- Principal idea: link **economically meaningful scenarios** to **correlation scenarios**
- **Stress testing**: portfolio effect of adverse correlation scenario
- **Reverse stress testing**: identify worst-case scenarios and their impact
- First application: correlation stress testing of **“London Whale”** portfolio

  Packham, N. and Woebbeking, F.: *A factor-model approach for correlation scenarios and correlation stress-testing*. Journal of Banking and Finance, 101 (2019), 92-103. link
- Current work: generalisation to **credit and stock portfolios**

Overview

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The "London Whale"

- **"London Whale"**: 2012 Loss at JPMorgan Chase & Co. of approx. 6.2 bn USD on a credit derivatives portfolio
- **Authorised trading position**, hence risk management problem
- **Synthetic credit portfolio (SCP)**: 120 long and short positions, **CDX** and **iTraxx** index + tranche products, investment grade and high-yield
- **"Smart short" strategy**: credit protection on high yield is financed by selling protection on investment grade indices.
- Timeline:
  - End of 2011: decision to reduce SCP's risk-weighted assets (RWA's).
  - Avoid liquidation costs by **increasing** positions with opposite market sensitivity (hedges).
  - 23 March 2012: Senior executives ordered to stop trading on SCP; net notional of 157 bn USD (up 260% from September 2011).
- **Risk management** of SCP focussed on **value-at-risk (VaR)** and **CSW-10** (credit spread widening of 10 basis points).
- Publicy available information: JPMorgan, 2013; United-States-Senate, 2013a,b

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The "London Whale" positions

Table: Top 10 Positions of SCP, 23 March 2012, USD net notional; several positions have a market share close to 50%.

Index					
Name	Series	Tenor	Tranche (%)	Protection	Net Notional ($)
CDX.IG	9	10yr	Untranched	Seller	72,772,508,000
	9	7yr	Untranched	Seller	32,783,985,000
	9	5yr	Untranched	Buyer	31,675,380,000
iTraxx.EU	9	5yr	Untranched	Seller	23,944,939,583
	9	10yr	22 – 100	Seller	21,083,785,713
	16	5yr	Untranched	Seller	19,220,289,557
CDX.IG	16	5yr	Untranched	Buyer	18,478,750,000
	9	10yr	30 – 100	Seller	18,132,248,430
	15	5yr	Untranched	Buyer	17,520,500,000
iTraxx.EU	9	10yr	Untranched	Seller	17,254,807,398
Net Total					137,517,933,681

Data source: United-States-Senate (2013a, Exhibit 36) and DTCC (2014, Section 1, Table 7).

Overview

Interest-rate modelling: Correlation parameterisation

- Parametric correlation models widespread in

  interest-rate modelling / LIBOR market model,

  e.g. Rebonato (2002); Brigo (2002); Schoenmakers and Coffey (2000); Packham (2005)

- Simplest case: Correlation c_{ij} between two forward LIBOR's is given by

$$c_{ij} = e^{-\beta|i-j|},$$

  where $\beta > 0$ is a parameter, and i, j represent maturities.

- Captures stylised fact that **correlations decay with increasing maturity difference**

Correlation parameterisation

- Idea: Carry over **"distance" measure** to other **risk factors**, such as geographic regions, industries, investment grade vs. high-yield, ...
- C: $n \times n$-correlation matrix of n financial instruments' returns.
- Factors that determine the correlations: $\mathbf{x} = (x^1, \ldots, x^m)'$.
- Correlation of securities i and j modelled as

$$c_{ij} = \exp(-(\beta_1 |x_i^1 - x_j^1| + \beta_2 |x_i^2 - x_j^2| + \cdots + \beta_m |x_i^m - x_j^m|), \qquad i, j = 1, \ldots, n,$$

  with $\beta_1, \ldots, \beta_m$ positive coefficients, determined through calibration.
- Functional form implies that the greater "distance" $|x_i^k - x_j^k|$, the greater de-correlation amongst securities i and j.
- If two instruments are identical in all respects, then correlation is 1.

Correlation parameterisation

- Given historical asset returns, parameters $\beta_1, \ldots, \beta_m$ are determined e.g. by OLS on transformed correlations $-\ln(c_{ij})$.
- **Scenario** (e.g. "the correlation between investment grade and high-yield securities decreases") is implemented by increasing corresponding β-parameter.
- With parameters calibrated on a regular basis, the parameter history can be used to obtain reasonable scenarios.

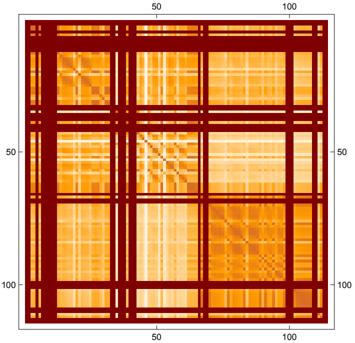
London whale: risk factors and correlation model

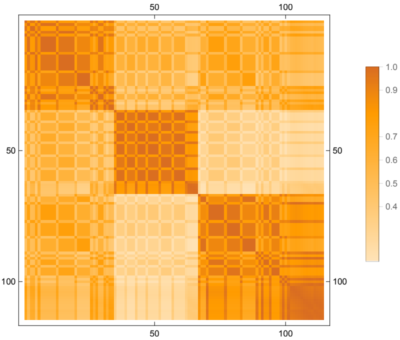
- All calculations on SCP portfolio of 23 March 2012 (117 instruments).
- Risk factors:
  - CDX vs. itraxx
  - investment grade vs. high yield
  - maturity
  - index series
  - index vs. tranche
- Parameterised correlation matrix:

$$c_{ij} = \exp\big(-(\beta_1|\mathsf{isCDX}_i - \mathsf{isCDX}_j| + \beta_2|\mathsf{isIG}_i - \mathsf{isIG}_j| + \beta_3|\mathsf{maturity}_i - \mathsf{maturity}_j| \\ + \beta_4|\mathsf{series}_i - \mathsf{series}_j| + \beta_5|\mathsf{isIndex}_i - \mathsf{isIndex}_j|)\big).$$

- Daily calibration of $\beta_1, \ldots, \beta_5$ from credit spread returns of 250 days.
- Time period: 1 March 2011 – 12 April 2012. Data source: Markit

London Whale: calibration and results





- Correlation matrices of 23 March 2012.
- Left: Empirical correlation matrix
- Right: parameterised (complete) correlation matrix
- Dark red entries: unavailable correlations
- Blocks of highly correlated data: CDX.IG, CDX.HY and iTraxx

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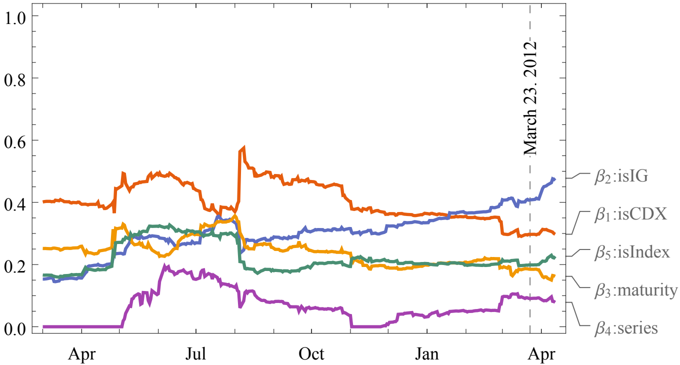

- Coefficients of CDX and itraxx positions in London Whale position; 01/03/2011–12/04/2012.
- Distances normalised to $[0, 1]$ to make coefficients comparable.

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Stress-testing correlations

- **Stress-test**: Effect on portfolio due to an adverse scenario.
- A shift in correlation has no *instantaneous* effect on portfolio value, therefore consider **portfolio risk**.
- Portfolio risk measured by **value-at-risk (VaR)** in variance-covariance approach:

$$\mathrm{VaR}_\alpha = -V_0 \cdot \mathrm{N}_{1-\alpha} \cdot \left(\mathbf{w}^\intercal\, \mathbf{\Sigma}\, \mathbf{w}\right)^{1/2},$$

  with
  - current position value V_0,
  - $\mathrm{N}_{1-\alpha}$: $(1-\alpha)$-quantile of the standard normal distribution,
  - vector of portfolio weights $\mathbf{w}$ and
  - covariance matrix $\mathbf{\Sigma}$.
- For **correlation stress test**, need to consider portfolio variance

$$\mathbf{w}^\intercal\, \mathbf{\Sigma}\, \mathbf{w}.$$

Core and peripheral risk factors*

- Following e.g. Kupiec (1998), **stress scenario** comprises
  - **"core"** risk factors (the ones that are stressed)
  - **"peripheral"** risk factors (affected by stress).
- $\boldsymbol{\beta}_s$: $j < m$ core factor parameters that are stressed directly
- $\boldsymbol{\beta}_u$: remaining $m - j$ peripheral risk factor parameters
- In **normal distribution setting**, optimal estimator of $\Delta\boldsymbol{\beta}_u$ conditional on $\Delta\boldsymbol{\beta}_s$:

$$\mathbb{E}(\Delta\boldsymbol{\beta}_u | \Delta\boldsymbol{\beta}_s) = \Sigma_{us}\Sigma_{ss}^{-1}\Delta\boldsymbol{\beta}_s,$$

where Σ_{us} and Σ_{ss} denote the covariance and variance matrices of $\boldsymbol{\beta}_u$ and $\boldsymbol{\beta}_s$.

Joint stress test of correlation and volatility*

- **Correlation shocks** often coincide with **volatility shocks**, see e.g. (Alexander and Sheedy, 2008; Longin and Solnik, 2001; Loretan and English, 2000).
- Simple model that combines both: **multivariate t-distribution**.
- In this case d-dimensional vector of asset returns $\mathbf{X}$ follows a **normal variance mixture distribution** with decomposition (e.g. Ch. 6.2 of McNeil *et al.* (2015))

$$\mathbf{X} = \sqrt{V} \cdot A \cdot \mathbf{Z},$$

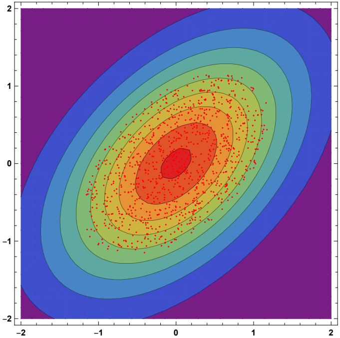
where
- $\mathbf{Z} \sim \mathrm{N}(0, I_k)$,
- V is a scalar r.v. independent of $\mathbf{Z}$,
- $V \sim \mathrm{Ig}(1/2\,\nu, 1/2\,\nu)$, i.e., V follows an inverse gamma distribution,
- A is a $d \times k$ matrix such that $\tilde{\mathbf{\Sigma}} = AA^T$.

Scenario selection and Mahalanobis distance

- **Scenario selection**: What is the worst scenario amongst all scenarios that occur within some pre-given probability?
- Let $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_m)^\mathsf{T}$ be a random vector with $\mathbb{E}(\boldsymbol{\beta}) = \overline{\boldsymbol{\beta}}$ and covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\beta}}$.
- **Mahalabonis distance**:

$$D(\boldsymbol{\beta}) = \left((\boldsymbol{\beta} - \overline{\boldsymbol{\beta}})^\mathsf{T} \boldsymbol{\Sigma}_{\boldsymbol{\beta}}^{-1} (\boldsymbol{\beta} - \overline{\boldsymbol{\beta}}) \right)^{1/2}.$$

- Maha associated with ellipsoids in normal (or elliptical) distributions.
- Find worst-case scenario within given ellipsoid.



Risk implications from correlation stress-testing

	correlation stress			plus vol stress	
Maha level	$\text{VaR}_{0.99}$	t-$\text{VaR}_{0.99}$	Change(%)	t-$\text{VaR}_{0.99}$	Change(%)
base case	339.32	354.98		354.98	
0.9	372.89	390.10	9.89	464.40	30.83
0.99	381.08	398.67	12.31	617.38	73.92
0.999	386.88	404.74	14.02	780.37	119.84
unconstrained*	620.96	649.62	83.00	1252.53	252.85

*Unconstrained w.r.t. correlation changes; vol stress level at 0.999.

- SCP portfolio's 1-day 99% value-at-risk for different Mahalanobis quantile constraints.
- Percentage changes denote relative distance to base VaR. For joint stress, percentage changes refer to base t-VaR scenario.
- t-distribution parameter ν calibrated to 13.5.
- Vol stress level for joint stress test is set to quantile in column one.

Risk-driver identification (reverse stress test)

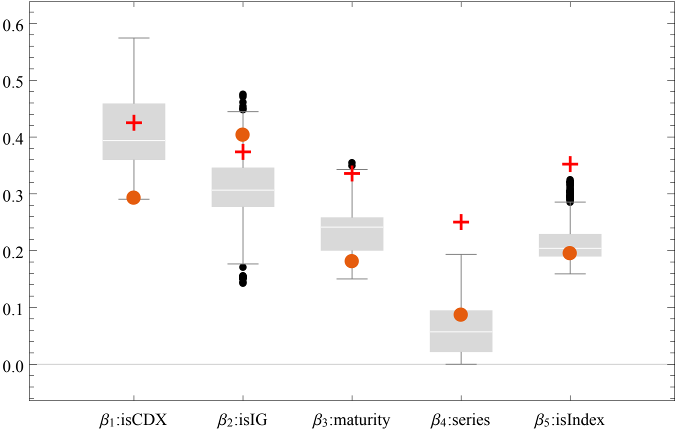


Figure: Box-plots of correlation parameters.

Dots: observed parameters as of 23.03.2012.

Crosses: worst-case scenario under a 99%-quantile Mahalanobis distance.

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Factor selection

- Risk factors in "London Whale" were tailored to specific portfolio.
- In practice, factor models use **industries** and **countries** as factors to model asset correlations.
- Problem: **How to assign factors to assets?**

Factor selection

- Risk factors in "London Whale" were tailored to specific portfolio.
- In practice, factor models use **industries** and **countries** as factors to model asset correlations.
- Problem: **How to assign factors to assets?**
- Number of factors should be **small**, but include all **important** factors.
- **Prior information**: country of firm's headquarter, primary industry
- $\rightsquigarrow$ **Bayesian variable selection** to determine small number of factors driving asset return

Link correlations to risk factors

- Association of asset $i \in \{1, \ldots, p\}$ with factor $k \in \{1, \ldots, d\}$:

$$\mathbf{1}_{\{k,i\}}$$

- Correlation parameterisation:

$$c_{ij} = \tanh\Big(\underbrace{\sum_{k=1}^{d} \lambda_k |\mathbf{1}_{\{k,i\}} - \mathbf{1}_{\{k,j\}}|}_{\text{"inter"-correlations}} + \underbrace{\sum_{k=1}^{d} \nu_k \mathbf{1}_{\{k,i\}} \mathbf{1}_{\{k,j\}}}_{\text{"intra"-correlations}} \Big),$$

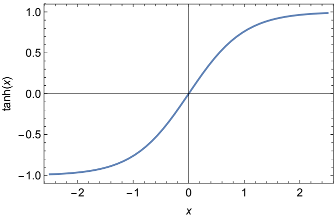
  with coefficients $\lambda_1, \ldots, \lambda_d, \nu_1, \ldots, \nu_d \in \mathbb{R}$.

Link correlations to risk factors

- $\tanh : \mathbb{R} \to [-1, 1]$ allows for negative correlations.
- $\tanh$ used in inferential statistics on sample correlation coefficients ($\rightsquigarrow$ Fisher transformation).

- The following summation formula is helpful for a rough interpretation of the coefficients:

$$\tanh(x+y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$$



Overview

Bayesian variable selection

- Different methods, e.g.
  - **Bayesian model selection** compares **posterior probabilities** of different models.
  - **Spike and slab priors** include an indicator variable for each coefficient and determines the indicator variable's **posterior probability** of taking value one.
- In our setting, **Bayesian model selection** worked best.

Bayesian model selection

- Denote candidate models by M_i, $i = 1, \ldots, m$.
- In a linear regression setting, each model M_i includes a specific subset of independent variables ($=$ potential risk factors) and excludes the other variables.
- **Posterior model probability**:

$$p(M_i|\boldsymbol{y}) \propto p(\boldsymbol{y}|M_i)p(M_i),$$

  where

  - $\boldsymbol{y}$ is the time series of a **firm's asset returns**,
  - $p(M_i)$ is the **prior model probability**,
  - $p(\boldsymbol{y}|M_i)$ is called the **marginal likelihood**.

  (see e.g. Appendix B.5.4 of (Fahrmeir *et al.*, 2013))

Bayesian model comparison

- **Posterior inclusion probabilities (PIP)**:

$$\mathbf{P}(\mathbf{1}_{\{\beta_k \neq 0\}} = 1|\boldsymbol{y}) = \sum_{\beta_k \in M_i} \mathbf{P}(M_i|\boldsymbol{y}).$$

- If number of parameters p is large, then full calculation of 2^p posterior model probabilities is infeasible.
- $\Rightarrow$ Use **Markov Chain Monte Carlo (MCMC)** simulation.

Example: VW

- Daily returns (2002-2018):
  - VW stock returns
  - MSCI stock indices; 11 industries and 24 countries as factors
- Factors with PIP greater 0.5 are selected:

```
>>> print(res[res['PIP']>0.5].round(4))
              coef     PIP     BVS  pvalue
4   MXWO0CD Index  1.0000  1.0000  0.0000
9   MXWO0TC Index  0.9848  0.9900  0.0017
10  MXWO0UT Index  0.9996  1.0000  0.0000
18         MSDUSZ  0.6788  0.4940  0.0105
19         MSDUAT  0.7998  0.7613  0.0000
34         MSDUGR  1.0000  1.0000  0.0000
```

- CD (Consumer Discretionary) and GR (Germany) have prior inclusion probability of 1.
- Other prior inclusion probabilities such that eight factors on average.

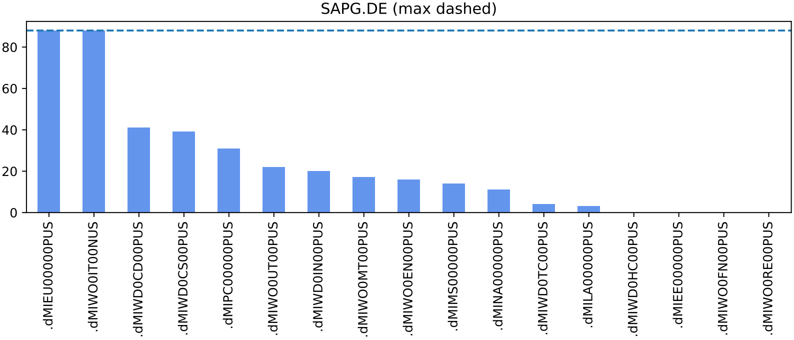
Overview

Factor selection

- Factors: MSCI stock indices representing 6 geographic regions and 11 industries
- Individual stocks: 500 S&P constituents, 30 DAX constituents
- Daily data from 1999-Jan 2021 (Source: Bloomberg, MSCI, Reuters)
- Factor assignment re-calibrated every quarter, based on 3-years of daily data (88 quarters)
- Prior: hard-code primary geographic region and industry
- 6 factors on expectation

Factor selection

- Number of quarters that each factor is included for SAP:



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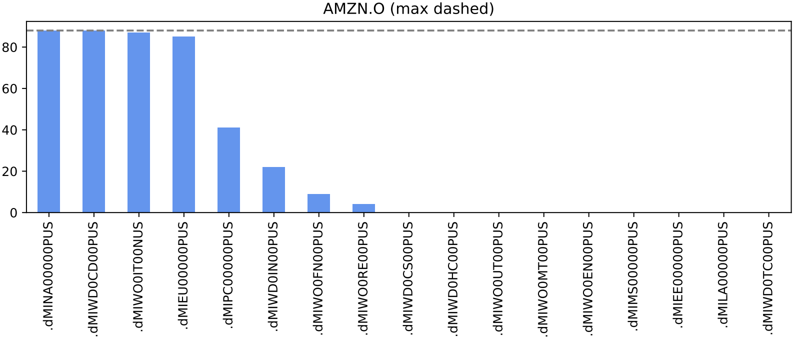
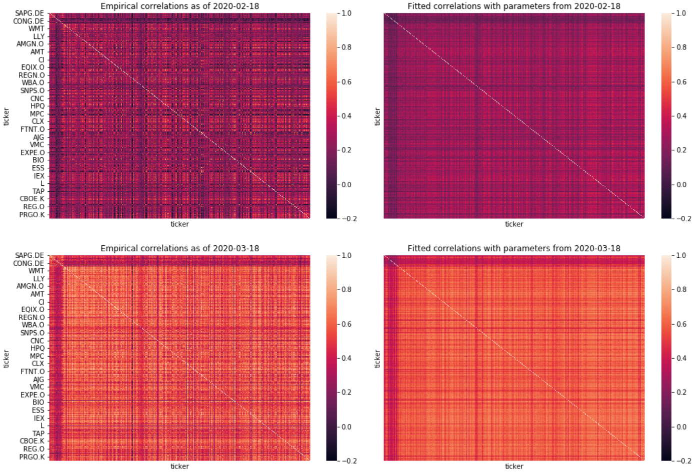
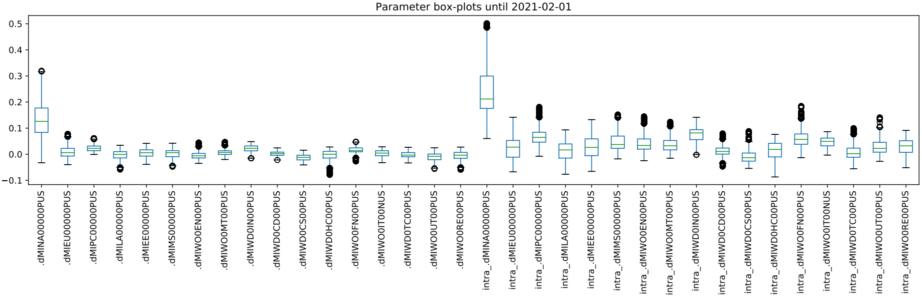


Correlations at beginning of Covid-19 pandemic



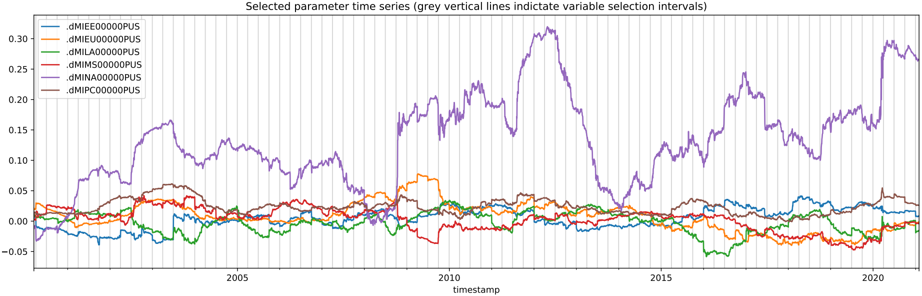
- Empirical & fitted correlations; top: 18 Feb, bottom: 18 Mar 2020.

Factor coefficients



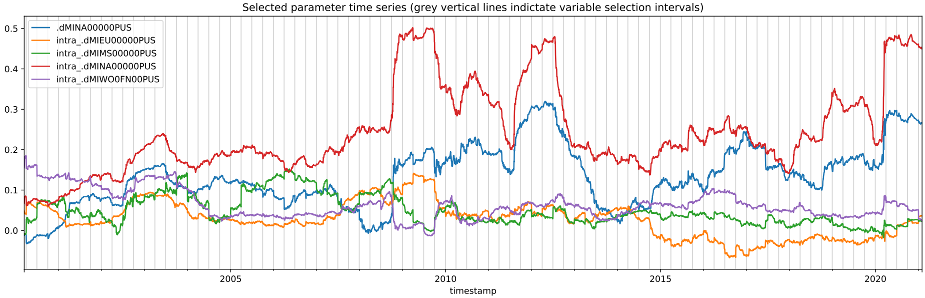
- Boxplots of coefficients of correlations between factors (left; "λ_k") and within factors (right; "ν_k").
- Intra-correlations are generally higher than inter-correlations.

Factor coefficients



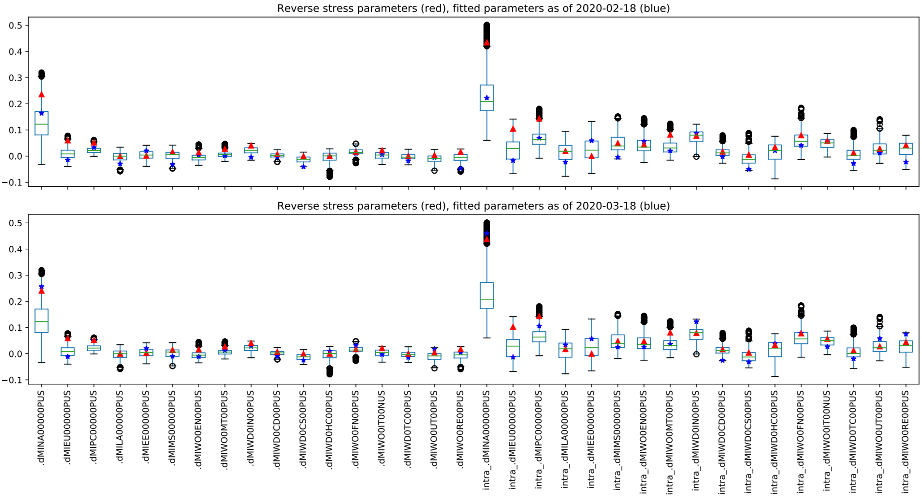
- Fitted "inter" parameters for selected risk factors ("λ_k").
- Blue: EM EMEA; orange: EU; green: EM L. Am.; red: EM Asia; purple: N. Am.; brown: Pacific

Factor coefficients



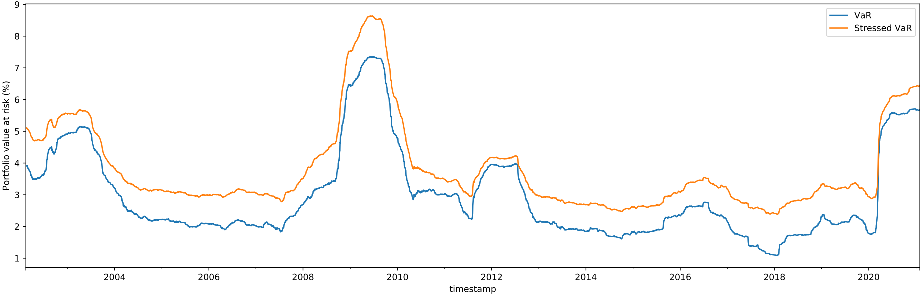
- Fitted "intra" parameters for selected risk factors ("ν_k").
- Blue: N. Am. inter; orange: EU; green: EM Asia; red: N. Am.; purple: Financials

Reverse stress testing (Covid-19 pandemic)



- Worst-case scenario within 99% Maha distance
- Partially realised in Feb/March 2020

Value-at-risk impact



- Blue: $\mathrm{VaR}_{99\%,1\ \mathrm{day}}$ on equally-weighted portfolio of DAX and S&P 500
- Orange: Stressed $\mathrm{VaR}_{99\%,1\ \mathrm{day}}$ on reverse stress scenario of 1 Feb 2021.

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- We develop a correlation stress testing framework, linking (risk) factors with correlations.
- Reverse stress tests can be conducted by assigning the factor loading a distribution.
- "London whale": a significant de-correlation between investment grade and high yield credit derivatives broke the "hedges" in the SCP.
- Simple correlation stress testing exposes the significant risks in a portfolio with high notional and low RWA.
- General case: factors (e.g. industries, countries) are linked firms via Bayesian variable selection methods
- Outlook: apply PCA to generate factors; factors can often be given an economic interpretation (global factor, Europe, etc.)

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Thank you!

Prof. Dr. Natalie Packham
Professor of Mathematics and Statistics
Berlin School of Economics and Law
Badensche Str. 52
10825 Berlin
natalie.packham@hwr-berlin.de